# Light Scalar Mesons within QCD sum rule 

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## Prologue:

## Study of dense matter using $\operatorname{SU}(2)$ Skyrme

 model with the trace anomaly of QCD- with B.-Y Park, D.-P. Min, M. Rho, V. Vento, 2002

$$
\begin{aligned}
\mathcal{L}= & \frac{f_{\pi}^{2}}{4}\left(\frac{\chi}{f_{\chi}}\right)^{2} \operatorname{Tr}\left(\partial_{\mu} U^{\dagger} \partial^{\mu} U\right)+\frac{1}{32 e^{2}} \operatorname{Tr}\left(\left[U^{\dagger} \partial_{\mu} U, U^{\dagger} \partial_{\nu} U\right]\right)^{2} \\
& +\frac{f_{\pi}^{2} m_{\pi}^{2}}{4}\left(\frac{\chi}{f_{\chi}}\right)^{3} \operatorname{Tr}\left(U+U^{\dagger}-2\right) \quad f_{\chi}: \text { VEV of } \chi \\
& +\frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi-\frac{1}{4} \frac{m_{\chi}^{2}}{f_{\chi}^{2}}\left[\chi^{4}\left(\ln \left(\chi / f_{\chi}\right)-\frac{1}{4}\right)+\frac{1}{4}\right] .
\end{aligned}
$$

- The scalar field $(\chi)$ with unity scale dim. and the potential energy generate the trace anomaly in this effective Lagrangian :


## Skyrmion Matter (SM) : Skyrmion Crystal

 * At low density : FCC (Face Centered Cubic)I

## Increasing matter density

Phase transition : chiral symm. restoration

* At high density :

Half Skyrmion CC (Cubic centered)

## Phases of skyrmion matter




Unit of density : $0.5 \sim \rho_{0}=0.17 \mathrm{fm}^{-3}$

- Pion decay constants for the massless pion as a function of density of SM


Unit of density : $0.5 \sim \rho_{0}=0.17 \mathrm{fm}^{-3}$

## In this talk

■ Short review of the QCD sum rule

■ Short review of the light scalar meson nonet

- QCD sum rule analysis of the light scalar nonet
- Summary


## QCD sum rule (SR)

- Correlator of the interpolating current $J_{S}$ with the quantum number of the hadron under consideration

$$
\Pi_{S}\left(q^{2}\right)=i \int_{\text {Nonperturbative QCD Vacuum }}^{i d^{4} x e^{i q \square x}\langle 0| T J_{S}(x) J_{S}^{?}(0)|0\rangle}
$$

- Calculating it in deeply Euclidean region by the perturbative OPE

$$
\Pi_{S}^{O P E}\left(q^{2}\right): \begin{aligned}
& \text { Condensates from the } \\
& \text { nonperturbative vacuum }
\end{aligned}
$$

- $\Pi^{\mathrm{OPE}}\left(q^{2}\right)$ is related to physical region by the dispersion relation

$$
\Pi_{S}^{O P E}\left(q^{2}\right)=\frac{1}{\pi} \int_{0}^{\infty} d s^{2} \frac{\operatorname{Im} \Pi_{S}\left(s^{2}\right)}{s^{2}-q^{2}}
$$

Narrow resonance approx. in the phen. side

$$
\langle 0| J_{S}|S\rangle=\sqrt{2} f_{S} M_{S}^{4} \quad \text { Quark-hadron duality }
$$

$$
\operatorname{Im} \Pi_{S}\left(s^{2}\right)=2 \pi f_{S}^{2} M_{S}^{8} \delta\left(s^{2}-M_{S}^{2}\right)+\theta\left(s^{2}-s_{0}^{2}\right) \operatorname{Im} \Pi_{S}^{O P E}\left(s^{2}\right)
$$

- $\operatorname{Im} \Pi_{s}\left(q^{2}\right)=\pi \sum_{n} \delta\left(q^{2}-m_{n}^{2}\right)\langle 0| J_{S}(0)|n\rangle\langle n| J_{S}^{2}(0)|0\rangle$
- Borel transform makes the contributions from the continuum suppressed exponentially.
- QCD sum rules :

$$
\begin{gathered}
\frac{1}{\pi} \int_{0}^{s_{0}^{2}} d s^{2} e^{-s^{2} / M^{2}} \operatorname{Im} \Pi_{S}^{O P E}\left(s^{2}\right)=2 f_{S}^{2} M_{S}^{8} e^{-M_{S}^{2} / M^{2}} \\
\tilde{\Pi}_{S}\left(M^{2}\right): \text { Must be POSITIVE }
\end{gathered}
$$

$M$ : Borel Mass

- Mass of Particle can be determined by

$$
M_{S}=\sqrt{\left(\partial_{M} \tilde{\Pi}_{S} / 2 \tilde{\Pi}_{S}\right) M^{3}}
$$

- Generally, including all contributions from OPE, the mass must be independent on the Borel mass.
- Actually, we cannot do it. Up to a certain energy dimension operators, mass plateau appears in some region of the Borel mass.
$\Longrightarrow$ Borel window
- Borel window must be opened in $M<s_{0}$.


## Light scalar meson nonet

- Members :

$$
\begin{aligned}
& I=1: a_{0}^{0}, a_{0}^{ \pm}(980) \\
& I=1 / 2: \kappa^{ \pm}, \kappa^{0}, \bar{\kappa}^{0}(800) \\
& I=0: \sigma(500), f_{0}(980)
\end{aligned}
$$

■ Large decay widths :

$$
\begin{gathered}
\Gamma_{a_{0}}=50 \sim 100 \mathrm{MeV}, \Gamma_{f_{0}}=40 \sim 100 \mathrm{MeV} \\
\Gamma_{\sigma}=400 \sim 700 \mathrm{MeV}
\end{gathered}
$$

Refs. : PDG, Chin. Phys. C, 38(2014) 09001

## $q \bar{q}$ interpretation

- With ideal mixing : $L=1$ for $P=+1$

$$
\begin{aligned}
& a_{0}^{+}(980)=u \bar{d}, a_{0}^{0}=\frac{1}{\sqrt{2}}(u \bar{u}-d \bar{d}), a_{0}^{-}=d \bar{u} \\
& \kappa^{+}(800)=u \bar{s}, \kappa^{0}=d \bar{s}, \bar{\kappa}^{0}=s \bar{d}, \kappa^{-}=s \bar{u} \\
& \sigma(600)=\frac{1}{\sqrt{2}}(u \bar{u}+d \bar{d}), f_{0}(980)=s \bar{s}
\end{aligned}
$$

- (?1)Decays of $a_{0}$ : fraction of $s \bar{s}$ ?

$$
\frac{\Gamma\left[a_{0}(980) \rightarrow \eta \pi\right]}{\Gamma\left[a_{0}(980) \rightarrow \eta \pi+K \bar{K}\right]}=0.85 \pm 0.02
$$

Amsler et al, Phys. Rep. 384(2004)61

■ (?2) Mass degeneracy in $a_{0}, f_{0}$

1. From number of strange quarks

$$
m_{f_{0}}>m_{\kappa}>m_{a_{0}}, m_{\sigma}
$$

2. $L=1$ gives 400 MeV more mass :
from the mass formula in a quark model
(Kochelev, H.-J. Lee, Vento, PLB 594 (2004) 87), for example : $f_{0}(980)$

$$
\begin{aligned}
& M_{f_{0}}=E_{\text {conf }}+2 m_{s}+E_{O G E}+E_{I}+E_{L=1} \\
& \quad \square 214+2 \times 407-2+0+400=1425 \mathrm{MeV}
\end{aligned}
$$

## [qq][ $\overline{q q}]$ interpretation

- One gluon exchange \& instanton: strongest attraction in two quarks of $\left|\overline{3}_{F}, \overline{3}_{C}, 1_{S}\right\rangle$ : scalar (S) diquark
in two antiquarks of $\left|3_{F}, 3_{C}, 1_{S}\right\rangle$ : S antidiquark - Jaffe \& Wilczek, Shuryak \& Zahed
- In flavor space :

Explicitly

$$
\begin{aligned}
& 3_{f} \otimes 3_{f}=\overline{3}_{A} \oplus 6_{S}, \overline{3}_{f} \otimes \overline{3}_{f}=3_{A} \oplus \overline{6}_{S} \\
& \Rightarrow \overline{3}_{A} \otimes 3_{A}=1 \oplus 8
\end{aligned}
$$

$$
\begin{aligned}
& {[u d]_{A} \leftrightarrow \bar{s},[u s]_{A} \leftrightarrow \bar{d},[d s]_{A} \leftrightarrow \bar{u}} \\
& {[\bar{u} \bar{d}]_{A} \leftrightarrow s,[\overline{u s}]_{A} \leftrightarrow d,[\bar{s}]_{A} \leftrightarrow u}
\end{aligned}
$$

- In terms of $S$ diquark \& $S$ antidiquark: $L=0$

$$
\begin{aligned}
& a_{0}^{+}(980)=[\bar{d} \bar{s}][u s], a_{0}^{0}=\frac{1}{\sqrt{2}}([\overline{d \bar{s}}][d s]-[\overline{u s}][u s]), a_{0}^{-}=[\overline{u s}][d s] \\
& \kappa^{+}(800)=[\bar{d} \bar{s}][u d], \kappa^{0}=[\overline{u s}][u d], \bar{\kappa}^{0}=[\bar{u} \bar{d}][u s], \kappa^{-}=[\bar{u} \bar{d}][d s] \\
& \sigma(600)=[\bar{u} \bar{d}][u d], f_{0}(980)=\frac{1}{\sqrt{2}}([\overline{d \bar{s}}][d s]+[\overline{u s}][u s])
\end{aligned}
$$

■ Number of strange quark:

$$
m_{f_{0}}=m_{a_{0}}>m_{\kappa}>m_{\sigma}: \text { Inverted mass spectrum }
$$

■ Strange quark component in $f_{0}, a_{0}$ :

$$
f_{0}, a_{0} \rightarrow K \bar{K}
$$

## SRs for light scalar nonet

■ Interpolating currents : energy dim. $=6$

$$
\begin{aligned}
J_{\sigma} & =\epsilon_{a b c} \epsilon_{a d e}\left(u_{b}^{T} C \gamma_{5} d_{d}\right)\left(\bar{u}_{d} C \gamma_{5} \bar{d}_{e}\right) \\
J_{f_{0}} & \left.=\frac{1}{\sqrt{2}} \epsilon_{a b c} \epsilon_{a d e}\left(u_{b}^{T} C \gamma_{5} s_{c}\right)\left(\bar{u}_{d} C \gamma_{5} \bar{s}_{e}\right)+(u \rightarrow d)\right) \\
J_{a_{0}^{o}} & =\frac{1}{\sqrt{2}} \epsilon_{a b c} \epsilon_{a d e}\left(\left(u_{b}^{T} C \gamma_{5} s_{c}\right)\left(\bar{u}_{d} C \gamma_{5} \bar{s}_{e}\right)-(u \rightarrow d)\right) \\
J_{\kappa^{+}} & =\epsilon_{a b c} \epsilon_{a d e}\left(u_{b}^{T} C \gamma_{5} d_{c}\right)\left(\bar{d}_{d} C \gamma_{5} \bar{s}_{e}\right)
\end{aligned}
$$

- After Borel transform :

Energy dimension of the correlator $=10$

- QCD SR : $m_{u}=m_{d}=0$ and up to $O\left(m_{s}\right)$



## LHS of SRs with scalar Diquark

■ Values of condensates and mass:

$$
\begin{gathered}
\langle\bar{u} u\rangle=-(0.25)^{3} \mathrm{GeV}^{3},\langle\bar{s} s\rangle=f_{s}\langle\bar{u} u\rangle,\left\langle g_{c}^{2} G^{2}\right\rangle=0.5 \mathrm{GeV}^{4}, \\
i g_{c}\langle\bar{u} \sigma \cdot G u\rangle=0.8 \mathrm{GeV}^{2}\langle\bar{u} u\rangle, i g_{c}\langle\bar{s} \sigma \cdot G s\rangle=f_{s} i g_{c}\langle\bar{u} \sigma \cdot G u\rangle, \\
m_{s}=0.15 \mathrm{GeV}, \quad f_{s}=0.8
\end{gathered}
$$


$f_{0}, a_{0}$

$\kappa$

$\sigma$

## What we have seen...

- SRs up to d=6 ops. :
- Multiquark system has large energy dim. : SRs up to $d=6$ ops. could not be enough.
- Large negative contribution from $d=8$ ops. : $\Longrightarrow$ destroys consistency of SRs.
- Effect from Instanton?
- Any possibility for keeping consistency of SRs: to kill large contributions from higher ops.?
- Generally, five types of relativistic currents:

$$
\begin{aligned}
& \overline{3}_{C} \otimes 3_{C}: J_{S}^{i}=\varepsilon_{a b c}\left[q_{1, b}^{T} \Gamma_{i}^{A} q_{2, c}\right] z_{a d e}\left[\bar{q}_{3, d}^{T} \bar{\Gamma}_{i}^{A} \bar{q}_{4, e}\right] \\
& 6_{C} \otimes \overline{6}_{c}: J_{S}^{i}=\left\{\left[q_{1, a}^{T} \Gamma_{i}^{s} q_{2, b}\right]+(a \leftrightarrow b)\right\}\left\{\left[\bar{q}_{3, a}^{T} \bar{\Gamma}_{i}^{s} \bar{q}_{4, b}\right]+(a \leftrightarrow b)\right\} \\
& \quad \text { with } \bar{\Gamma}=\gamma_{0} \Gamma^{2} \gamma_{0}, \text { and } \Gamma_{i}^{A T}=-\Gamma_{i}^{A}, \Gamma_{i}^{s, T}=\Gamma_{i}^{S} \\
& \Longrightarrow \quad \begin{array}{l}
\Gamma_{i}^{A}=C \gamma_{5}(S), C(P S), C \gamma_{5} \gamma_{\mu}(V) \\
\Gamma_{i}^{S}=C \gamma_{\mu}(A V), C \sigma_{\mu \nu}(T)
\end{array}
\end{aligned}
$$

- General interpolating currents :

$$
J_{S}=\alpha J_{S}^{S}+\beta J_{S}^{P S}+v J_{S}^{V}+v^{\prime} J_{S}^{A V}+t J_{S}^{T}
$$

Chen et al., Phys.Lett.B650:369-372,2007

## SR for sigma again

- 't Hooft instanton induced interaction for $u, d$ :

$$
\mathcal{L}=\frac{G}{4\left(N_{c}^{2}-1\right)}\left[\frac{2 N_{c}-1}{2 N_{c}}\left(\left(\bar{\psi} \tau_{\alpha}^{-} \psi\right)^{2}+\left(\bar{\psi} \gamma_{5} \tau_{\alpha}^{-} \psi\right)^{2}\right)+\frac{1}{4 N_{c}}\left(\bar{\psi} \sigma_{\rho \sigma} \tau_{\alpha}^{-} \psi\right)^{2}\right]
$$

Fierz trans.

$$
\begin{aligned}
& \mathcal{L}= \frac{G}{2 N_{c}\left(N_{c}-1\right)} \epsilon_{a b c} \epsilon_{a d e}\left[\left(u_{b}^{T} \Gamma_{S} d_{c}\right)\left(\bar{u}_{d} \Gamma_{S} \vec{d}_{e}^{T}\right)-\left(u_{b}^{T} \Gamma_{P S} d_{c}\right)\left(\bar{u}_{d} \Gamma_{P S} \bar{d}_{e}^{T}\right)\right] \\
&+\frac{G}{4 N_{c}\left(N_{c}+1\right)}\left(u_{a}^{T} \Gamma_{T, \rho \sigma} d_{a^{\prime}}\right)\left(\left(\bar{u}_{a} \bar{\Gamma}_{T}^{\rho \sigma} \bar{d}_{a^{\prime}}^{T}\right)+\left(\bar{u}_{a^{\prime}} \bar{\Gamma}_{T}^{\rho \sigma} \bar{d}_{a}^{T}\right)\right), \\
& \alpha=1, \beta=-1, v=0, v^{\prime}=0, t=1 / 4 \text { for } N_{C}=3
\end{aligned}
$$

- From PDG:


## $f_{0}(500)$ DECAY MODES

|  | Mode | Fraction $\left(\Gamma_{i} / \Gamma\right)$ |
| :--- | :--- | :--- |
| $\Gamma_{1}$ | $\pi \pi$ | dominant |
| $\Gamma_{2}$ | $\gamma \gamma$ | seen |

- Interpolating current of the tetraquark can couple to the two pion state : Fierz transf.

■ We need to modity the phenomenological side.

- $\operatorname{Im} \Pi_{S}\left(q^{2}\right)=\pi \sum_{n} \delta\left(q^{2}-m_{n}^{2}\right)\langle 0| J_{S}(0)|n\rangle\langle n| J_{S}^{?}(0)|0\rangle$
- Narrow resonance + two pion state in the phen. side :

- PCAC gives :

$$
\begin{aligned}
\frac{1}{\pi} \Pi^{2 \pi}\left(q^{2}\right)= & \frac{6}{16^{2} \pi^{2}}\left[(\alpha-\beta)^{2}\left(\frac{\langle\bar{q} q\rangle^{2}}{4 f_{\pi}^{2}}\right)^{2}+(\alpha+\beta)^{2}\left(\frac{f_{\pi}^{2}}{4}\right)\left(q^{2}-2 m_{\pi}^{2}\right)^{2}\right] \\
& \times \sqrt{1-\frac{4 m_{\pi}^{2}}{q^{2}}} \theta\left(q^{2}-4 m_{\pi}^{2}\right)
\end{aligned}
$$

- Instanton effects :


$$
\begin{aligned}
& \Pi^{I+\bar{I}}(q)=\left(\alpha^{2}-\beta^{2}\right) \frac{32 n_{\mathrm{eff}} \rho_{c}^{4}}{\pi^{8} m_{q}^{* 2}} f_{6}(q) \\
& +\left[19\left(\alpha^{2}+\beta^{2}\right)-6 \alpha \beta\right] \frac{n_{\mathrm{eff}} \rho_{c}^{4}\langle\bar{q} q\rangle^{2}}{18 \pi^{4} m_{q}^{* 2}} f_{0}(q)
\end{aligned}
$$

- QCD sum rules :

$$
\begin{aligned}
& \frac{1}{\pi} \int_{0}^{s_{0}^{2}} d s^{2} e^{-s^{2} / M^{2}} \operatorname{Im} \Pi^{O P E}\left(s^{2}\right)+\hat{B}\left[\Pi^{I+\bar{I}}(q)\right]-\frac{1}{\pi} \int_{4 m_{\pi}^{2}}^{s_{0}^{2}} d s^{2} e^{-s^{2} / M^{2}} \operatorname{Im} \Pi^{2 \pi}\left(s^{2}\right) \\
& =2 f_{f_{0}}^{2} m_{f_{0}}^{8} e^{-m_{f_{0}}^{2} / M^{2}}, \\
& \frac{1}{\pi} \operatorname{Im} \Pi^{\mathrm{OPE}}\left(q^{2}\right)
\end{aligned} \quad=\left(\alpha^{2}+\beta^{2}\right)\left[\frac{\left(q^{2}\right)^{4}}{2^{12} \cdot 5 \cdot 3 \pi^{6}}+\frac{\left\langle g^{2} G^{2}\right\rangle}{2^{11} \cdot 3 \pi^{6}}\left(q^{2}\right)^{2}\right] .
$$

- Including the form factor, for $\mathrm{a}=-\mathrm{b}=1$, there can be stahla raciltl


FIG. 7. The mass obtained from the SR for $\alpha=-\beta=1$ FIG. 8. The contribut including the effect of the form factor as a function of the shown: direct instantor Borel parameter. The dashed line corresponds to the mass dashed line), and OPE ' obtained from the SR not including the two pion contribution.

## Other members with diquarks

## - For f0(980) and a0(980)

$$
L_{f_{0}, a_{0}}^{O P E}(M)=\left(\alpha^{2}+\beta^{2}\right)\left(\frac{M^{10} E_{4}}{2^{9} \cdot 5 \pi^{6}}+\frac{g^{2}\left\langle G^{2}\right\rangle M^{6} E_{2}}{2^{10} \cdot 3 \pi^{6}}+\frac{m_{s}\langle\bar{s} s\rangle M^{6} E_{2}}{2^{5} \cdot 3 \pi^{4}}+\frac{m_{s} i g\langle\bar{s} \sigma \cdot G s\rangle M^{4} E_{1}}{2^{7} \cdot 3 \pi^{4}}\right.
$$

$$
\left.+\frac{m_{s} g^{2}\left\langle G^{2}\right\rangle\langle\bar{s} s\rangle M^{2} E_{0}}{2^{8} \cdot 3 \pi^{4}}-\frac{m_{s}\langle\bar{q} q\rangle^{2}\langle\bar{s} s\rangle}{9}\right)-\left(\alpha^{2}-\beta^{2}\right)\left(\frac{m_{s}\langle\bar{q} q\rangle M^{6} E_{2}}{2^{4} \cdot 3 \pi^{4}}-\frac{\langle\bar{q} q\rangle\langle\bar{s} s\rangle M^{4} E_{1}}{12 \pi^{2}}\right.
$$

$$
-\frac{m_{s} i g\langle\bar{q} \sigma \cdot G q\rangle M^{4}}{2^{6} \pi^{4}}\left(E_{1}+\bar{W}_{1}\right)+\frac{M^{2} E_{0}}{2^{3} \cdot 3 \pi^{2}}(\langle\bar{q} q\rangle i g\langle\bar{s} \sigma \cdot G s\rangle+\langle\bar{s} s\rangle i g\langle\bar{q} \sigma \cdot G q\rangle)
$$

$$
+\frac{m_{s} g^{2}\left\langle G^{2}\right\rangle\langle\bar{q} q\rangle}{2^{7} \cdot 9 \pi^{4}}\left(5 E_{0}+6 W_{0}\right)-59 \frac{i g\langle\bar{q} \sigma \cdot G q\rangle i g\langle\bar{s} \sigma \cdot G s\rangle}{2^{10} \cdot 9 \pi^{2}}
$$

$$
\left.-7 \frac{g^{2}\left\langle G^{2}\right\rangle\langle\bar{q} q\rangle\langle\bar{s} s\rangle}{2^{6} \cdot 3^{3} \pi^{2}}-\frac{m_{s}\langle\bar{q} q\rangle\langle\bar{s} s\rangle^{2}}{18}\right)
$$

$$
\begin{aligned}
& J_{J, \ldots,}^{s, ~} \\
& L_{f_{0}, a_{0}}^{I n s t}(M)=\left(\alpha^{2}-\beta^{2}\right) \frac{32 n_{e f f} \rho_{c}^{4}}{\pi^{8} m_{q}^{*} m_{s}^{*}} \hat{B}\left[I_{6}(Q)\right]+\left(19 \alpha^{2}+19 \beta^{2}-6 \alpha \beta\right) \frac{n_{e f f} \rho_{c}^{4}\langle\bar{q} q\rangle\langle\bar{s} s\rangle}{18 \pi^{4} m_{q}^{*} m_{s}^{*}} \hat{B}\left[I_{0}(Q)\right] \\
& \mp(\alpha-\beta)^{2} \frac{n_{e f f} \rho_{c}^{4}\langle\bar{s} s\rangle^{2}}{12 \pi^{4} m_{q}^{*} 2} \hat{B}\left[I_{0}(Q)\right] \pm(\alpha-\beta)^{2} \frac{8 n_{e f f} \rho_{c}^{6}\langle\bar{s} s\rangle}{3 \pi^{6} m_{q}^{*} 2 m_{s}^{*}} \hat{B}\left[g_{0}(Q)\right] . \\
& \text { Upper sign : f0(980) }
\end{aligned}
$$

- Mass degeneracy in $\mathrm{f} 0(980)$ and $\mathrm{a} 0(980)$

$$
\alpha=\beta
$$

- Mass fitting


Mass of $f_{0}(980), a_{0}(980)$ from the QCD sum rule with $s_{0}=1.37 \mathrm{GeV}$.

## ■ For kappa(800)

$$
\begin{aligned}
& L_{\kappa}^{O P E}(M)=\left(\alpha^{2}+\beta^{2}\right)\left(\frac{M^{10} E_{4}}{2^{9} \cdot 5 \pi^{6}}+\frac{g^{2}\left\langle G^{2}\right\rangle M^{6} E_{2}}{2^{10} \cdot 3 \pi^{6}}+\frac{m_{s}\langle\bar{s} s\rangle M^{6} E_{2}}{2^{6} \cdot 3 \pi^{4}}+\frac{m_{s} i g\langle\bar{s} \sigma \cdot G s\rangle M^{4} E_{1}}{2^{8} \cdot 3 \pi^{4}}\right. \\
& \left.+\frac{m_{s} g^{2}\left\langle G^{2}\right\rangle\langle\bar{s} s\rangle M^{2} E_{0}}{2^{9} \cdot 3 \pi^{4}}-\frac{m_{s}\langle\bar{q} q\rangle^{3}}{18}\right)-\left(\alpha^{2}-\beta^{2}\right)\left(\frac{m_{s}\langle\bar{q} q\rangle m^{6} E_{2}}{2^{5} \cdot 3 \pi^{4}}-\frac{\langle\bar{q} q\rangle(\langle\bar{q} q\rangle+\langle\bar{s} s\rangle) M^{4} E_{1}}{24 \pi^{2}}\right. \\
& -\frac{m_{s} i g\langle\bar{q} \sigma \cdot G q\rangle M^{4}}{n^{7}}\left(E_{1}+\bar{W}_{1}\right)+\frac{M^{2} E_{0}}{2}(\langle\bar{q} q\rangle i g\langle\bar{s} \sigma \cdot G s\rangle+\langle\bar{s} s\rangle i g\langle\bar{q} \sigma \cdot G q\rangle+2\langle\bar{q} q\rangle i g\langle\bar{q} \sigma \cdot G q\rangle)
\end{aligned}
$$

$$
\begin{aligned}
& L_{\kappa}^{\text {Inst }}(M)=\left(\alpha^{2}-\beta^{2}\right) \frac{16 n_{e f f} \rho_{c}^{4}}{\pi^{8} m_{q}^{* 2}}\left(1+\frac{m_{q}^{*}}{m_{s}^{*}}\right) \hat{B}\left[I_{6}(Q)\right]+\left(\alpha^{2}+\beta^{2}\right) \frac{n_{e f f} \rho_{c}^{4}\langle\bar{q} q\rangle}{36 \pi^{4} m_{q}^{* 2}}\left(19\langle\bar{s} s\rangle+22 \frac{m_{q}^{*}}{m_{s}^{*}}\langle\bar{q} q\rangle\right) \hat{B}\left[I_{0}(Q)\right] \\
& -\alpha \beta \frac{n_{e f f} \rho_{c}^{4}\langle\bar{q} q\rangle}{6 \pi^{4} m_{q}^{* 2}}\left(\langle\bar{s} s\rangle+2 \frac{m_{q}^{*}}{m_{s}^{*}}\langle\bar{q} q\rangle\right) \hat{B}\left[I_{0}(Q)\right]-(\alpha-\beta)^{2} \frac{8 n_{e f f} \rho_{c}^{6}\langle\bar{q} q\rangle}{3 \pi^{6} m_{q}^{* 2} m_{s}^{*}} \hat{B}\left[g_{0}(Q)\right]
\end{aligned}
$$

- With $\alpha=\beta$, mass fitting :


Mass of $\kappa$ as a function of $M$ with $s_{0}=1.43 \mathrm{GeV}$

## Bound state of two psedoscalar mesons?

- For f0(980) : bound state of two etas?
Y.U. Surovtsev et al., Int. J. Mod. Phys. A 26, 610 (2011)
-From analysis of resonances appearing in

$$
\begin{aligned}
& \pi \pi \rightarrow \pi \pi, K \bar{K}, \eta \eta \\
& J / \psi \rightarrow \pi \pi, K \bar{K}
\end{aligned}
$$

- Interpolating current :

$$
\begin{gathered}
J=J_{\eta} J_{\eta}=\alpha^{2} J_{8} J_{8}+2 \alpha \beta J_{8} J_{1}+\beta^{2} J_{1} J_{1} \\
J_{8}=i\left(\bar{u} \gamma_{5} u+\bar{d} \gamma_{5} d-2 \bar{s} \gamma_{5} s\right), J_{1}=i\left(\bar{u} \gamma_{5} u+\bar{d} \gamma_{5} d+\bar{s} \gamma_{5} s\right) \\
\left|\begin{array}{l}
\theta_{p}=-11.5^{\circ} \\
\psi_{8}=u \bar{u}+d \bar{d}-2 s \bar{s}, \quad \psi_{1}=u \bar{u}+d \bar{d}+s \bar{s}
\end{array}\right|
\end{gathered}
$$

## - Left Hand side of SR


$+\left(3 L^{2} 2(c+s)^{\sim}(2 C-s)^{\sim}-\left(3(2 C-s)^{ \pm}\right) \frac{-}{2^{1+1} \cdot 3^{2} \pi^{2}}\right.$

$$
-4(c+s)^{2}(2 c-s)^{2} \frac{m_{s}\langle\bar{q} q\rangle^{2}\langle\bar{s} s\rangle}{12}-13(2 c-s)^{4} \frac{m_{s}\langle\bar{s} s\rangle^{3}}{72}
$$

## Contributions from the instanton:



## Another possibility :

- For f0(980): bound state of two Kaons?
- Weinstein and Isgur, PRL 48, 659 (1982), PRD 27, 588 (1983) : Using the color hyperfine and harmonic oscillator potentials.
- T. Branz, et. Al. , Eur. Phys. J. A 37, 303 (2008)
: Using a phenomenological Lagrangian.

■ Interpolating current :

$$
\begin{aligned}
& \left|f_{0}(980)\right\rangle=\alpha\left|K^{+} K^{-}\right\rangle+\beta\left|K^{0} \bar{K}^{0}\right\rangle \\
J_{f_{0}} & =\alpha J_{K^{+}} J_{K^{-}}+\beta J_{K^{0}} J_{\bar{K}^{0}} \\
& =-\left[\alpha\left(\bar{s} \gamma_{5} u\right)\left(\bar{u} \gamma_{5} s\right)+\beta\left(\bar{s} \gamma_{5} d\right)\left(\bar{d} \gamma_{5} s\right)\right]
\end{aligned}
$$

## - Left hand side of SR :



$I^{6} E_{2}\left(M^{2}\right)$
$\left(M^{2}\right)$
.



## Contributions from the instantons:



## Summary

- For sigma(500) : it could be a diquark-antidiquark bound state. Effect from width?
- Are other members diquark-antidiquark bound states? - Mass splitting from sigma is too small even though they have strange quark.
- Can f0(980) be a bound state of two mesons? - we did not see a signal which $\mathrm{f0}(980)$ is a bound state of the two etas or the two kaons.
- Mixing tetraquarks and two quark state, or glueballs...
- Instaton induced interaction in three flavors could give a hint for understanding the scalar mesons.

Thank you!

